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TRANSIENT HEAT FLOW BETWEEN CONTACTING SOLIDS

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Abstract—Approximate solutions valid for different time domains are found for the problem of transient heat flow between contacting solids. The parameters governing the heat flow and the applicability of the different solutions are given in terms of the contact geometry.

NOMENCLATURE

A, area, cm^2;
A_0, area per contact spot, cm^2;
a, contact spot radius, cm;
b, characteristic dimension, cm;
d, characteristic dimension, cm;
H, thermal conductance, cal/cm^2 degC s;
K, thermal conductivity, cal/cm degC s;
N, contact spot density, cm^{-2};
q, heat flux, cal/cm^2 s;
q', heat flux per contact spot, cal/cm^2 s;
R, thermal resistance, degC cm^2 s/cal;
s, Laplace transform variable, s^{-1};
T, temperature, °C;
u, dimensional constant, cm^{-1};
v, dimensional constant, cm^{-1};
V, temperature, °C.

Greek symbols

α, thermal diffusivity, cm^2/s;
β, defined by equation (7), s$^{-1/2}$;
γ, dimensionless constant;
δ, contact spot spacing, cm;
ρ, thermal resistance per contact spot, degC s/cal;
ω, defined by equation (45), cm.

INTRODUCTION

THE problem of steady-state conduction across the interface between two solids in mechanical contact has been examined in increasing detail [1, 2, 3]. Less attention has been paid to the transient case which occurs in frictional heating, in the molding or shaping of materials whose temperatures differ from that of the tool and in the heating or cooling of laminated bodies. The results presented here were developed during a study of heat transfer in glass forming, wherein heat is extracted from the glass by a colder mold in order to increase the viscosity of the glass to the point where it is essentially rigid.

It has been well established that the surfaces of solid bodies which are pressed together actually touch only at isolated spots and that the true contact area is a small fraction of the total area [4]. Thus, the heat (or electrical) flow between such bodies is in part confined to the spots, resulting in converging and diverging flow at each spot. Heat and usually to a much lesser extent, electricity, can also flow across the gap which exists between the contact spots provided the material in the gap is a conductor or the temperature level is great enough for radiation to play an important role. The convergence at the spots results in an effective contact resistance per spot, ρ, for steady state flow given by [4, 5, 6]:

$$\rho = \frac{1}{4a\,K_1} + \frac{1}{4a\,K_2} \tag{1}$$

where ρ is defined by

$$q'\,\pi a^2 = \frac{\Delta T}{\rho}. \tag{2}$$

a is the radius of the spot and the K's are the conductivities of the two materials. If the density

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of spots is N, the overall resistance, R, per unit area, is

$$R = \frac{1}{4aN}\,\frac{K_1 + K_2}{K_1\,K_2}. \qquad (3)$$

In heat-transfer work this is usually replaced by its inverse the surface conductance, H:

$$H = \frac{1}{R} = \frac{4aNK_1K_2}{K_1 + K_2}. \qquad (3a)$$

The quantity $2K_1\,K_2/(K_1 + K_2)$ is the harmonic mean of the thermal conductivities. In order to determine the transient flow of heat between two bodies, the case of an isolated contact spot is first examined, followed by the case of many spots.

TRANSIENT FLOW THROUGH A SINGLE-CONTACT SPOT

Figure 1 shows the cross section of a contact spot of radius a between two bodies with different properties, denoted by the subscripts 1 and 2, and different initial temperatures, T_1 and T_2. The two bodies are taken to be semi-infin·te in extent.

The small volume of material inside the spherical surface of radius a is assumed to be a perfect conductor and the heat flow outside of the sphere is assumed to be radial only, which is certainly true at some distance from the spot.

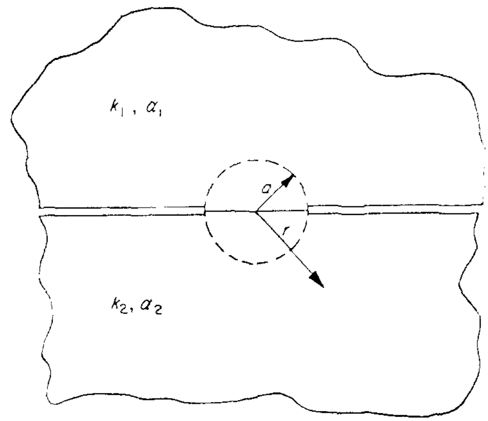

FIG. 1.

The temperature distribution $T(r, t)$, is then the solution of

$$\left.\begin{aligned} \frac{1}{r}\,\frac{\partial^2}{\partial r^2}(r\,V_1) &= \frac{1}{\alpha_1}\,\frac{\partial V_1}{\partial t} \\ \frac{1}{r}\,\frac{\partial^2}{\partial r^2}(r\,V_2) &= \frac{1}{\alpha_2}\,\frac{\partial V_2}{\partial t} \end{aligned}\right\}\ r > a \qquad (4)$$

with the boundary conditions

$$\left.\begin{aligned} &\left.\begin{aligned} V_1 &= V_2 \\ K_1\,\frac{\partial V_1}{\partial r} &= -\,K_2\,\frac{\partial V_2}{\partial r} \end{aligned}\right\}\ r = a, t > 0 \\ &\left.\begin{aligned} V_1 &= T_1 \\ V_2 &= T_2 \end{aligned}\right\}\ t = 0 \end{aligned}\right\}. \qquad (5)$$

Using the Laplace transform method [6, 7], it is found that the transforms are

$$V_1(r, s) = T_1/s + (T_2 - T_1)\,\frac{K_2\,\alpha_2^{-1/2}\,[(\sqrt{s}) + \alpha_2/a]\,\exp\,[-\,(r-a)\,\sqrt{(s/\alpha_1)}]}{(K_2\,\alpha_2^{-1/2} + K_1\,\alpha_1^{-1/2})\,s\,(r/a)\left[(\sqrt{s}) + \dfrac{(K_1 + K_2)}{a\,(K_1\,\alpha_1^{-1/2} + K_2\,\alpha_2^{-1/2})}\right]} \qquad (6)$$

and a similar equation for $V_2(r, s)$ found by interchanging subscripts. Letting

$$\beta = \frac{K_1 + K_2}{(K_1\,\alpha_1^{-1/2} + K_2\,\alpha_2^{-1/2})\,a} \qquad (7)$$

from a table of transforms the inverse of (6) is

$$\frac{V_1 - T_1}{T_2 - T_1} = \frac{K_2\,\alpha_2^{-1/2}\left(\dfrac{a}{r}\right)}{K_1\,\alpha_1^{-1/2} + K_2\,\alpha_2^{-1/2}}\left\{\exp\,[\beta^2\,t + \beta\,(r-a)/\sqrt{\alpha_1}]\,\mathrm{erfc}\left[\beta\,(\sqrt{t}) + \frac{r-a}{2\sqrt{(\alpha_1\,t)}}\right]\right.$$
$$\left. + \frac{\sqrt{\alpha_2}}{a\beta}\left[\mathrm{erfc}\,\frac{(r-a)}{2\sqrt{(\alpha_1\,t)}} - \exp\,[\beta^2\,t + \beta\,(r-a)/\sqrt{\alpha_1}]\,\mathrm{erfc}\left[\beta\,(\sqrt{t}) + \frac{r-a}{2\sqrt{(\alpha_1\,t)}}\right]\right]\right\}. \qquad (8)$$

At $t > 1/\beta^2$, the contact temperature, $V(a, t)$ at $r = a$ approaches the steady-state value

$$\frac{V(a, \infty) - T_1}{T_2 - T_1} = \frac{K_2}{K_1 + K_2}. \tag{9}$$

This result can be found directly from the transform without inverting by multiplying the transform by s and letting s approach zero. If the two bodies have the same thermal properties, the steady-state contact temperature, which is the mean of the initial temperatures, is reached instantaneously since the terms with exponentials drop out. The transform of the heat flux through the spot is

$$q'(a, s) = \frac{(T_2 - T_1) K_1 \left[(\sqrt{s}) + \frac{\sqrt{\alpha_1}}{a}\right]}{s(\sqrt{\alpha_1}) \left\{ \frac{K_1(\sqrt{\alpha_2}) \left[(\sqrt{s}) + \frac{\ }{a}\right]}{K_2(\sqrt{\alpha_1}) \left[(\sqrt{s}) + \frac{\sqrt{\alpha_1}}{a}\right]} + 1 \right\}} \tag{10}$$

which reaches a steady-state value of

$$q'(a, \infty) = (T_2 - T_1) \frac{K_1 K_2}{a(K_1 + K_2)} \tag{11}$$

The resistance is then

$$R = \frac{T_2 - T_1}{\pi a^2 q} = \frac{K_1 + K_2}{\pi a K_1 K_2} \tag{12}$$

agreeing with (3) quoted above except for the factor $\pi/4$, which results from treating the constriction as a spherical rather than a circular surface. The inverse of (10) is:

$$\left.\begin{aligned} q'(a, t) = (T_2 - T_1) \frac{K_1 K_2 \alpha_1^{-1/2} \alpha_2^{-1/2}}{K_1 \alpha_1^{-1/2} + K_2 \alpha_2^{-1/2}} \\ \left[\frac{1}{\sqrt{(\pi t)}} + \frac{\sqrt{(\alpha_1 \alpha_2)}}{a^2 \beta} - \beta \left(1 - \frac{\sqrt{\alpha_1}}{\beta a}\right)\right. \\ \left.\left(1 - \frac{\sqrt{\alpha_2}}{\beta a}\right) \exp(\beta^2 t) \operatorname{erfc} \beta \sqrt{t}\right]. \end{aligned}\right\} \tag{13}$$

If the two materials have the same properties, (13) becomes

$$q'(a, t) = (T_1 - T_2) \left[\frac{K}{2\sqrt{(\pi \alpha t)}} + \frac{K}{2a}\right]. \tag{13a}$$

The term in (13) containing the exponential can be neglected if the thermal diffusivities do not differ greatly. The remaining time-dependent term becomes negligible when

$$t > a^2/\pi\alpha.$$

With a contact spot diameter of 0·01 cm [4] and typical metal properties;

$$a^2/\pi\alpha \simeq 5 \times 10^{-4} \text{ s}.$$

The sphere of the same radius which has been assumed to be a perfect conductor also has a response time of this order [8] so that the overall response time is of the same order.

The initial heat flux is thus infinite. The first term in (13) is the solution of problem of two semi-infinite solids with perfect contact so that the remaining terms are due to the convergence at the spot. For the bodies as a whole, immediately after contact, the heat flux is given by

$$q = (T_1 - T_2) \frac{K_1 K_2 \alpha_1^{-1/2} \alpha_2^{-1/2} \pi a^2 N}{(K_1 \alpha_1^{-1/2} + K_2 \alpha_2^{-1/2})} \left[\frac{1}{\sqrt{(\pi t)}} + \frac{\sqrt{(\alpha_1 \alpha_2)}}{a^2 \beta}\right]. \tag{14}$$

The conclusion is that for $t < (a^4 \beta^2/\alpha_1 \alpha_2 \pi)$, the bulk average temperatures at a distance perpendicular to the plane of contact respond as if the initial temperature difference were reduced by the factor $\pi a^2 N$, i.e. the fractional surface contact area, and no contact resistance exists.

In real bodies the steady-state solution (11) will not be reached, since after some time the temperature fields due to neighboring contact spots overlap. This effect is investigated next.

RESPONSE OF AN ARRAY OF CONTACT SPOTS

Assuming a uniform density, N, of contact spots and a uniform contact spot area A, for semi-infinite bodies, the flux through a contact spot is $(NA_0)^{-1}$ times higher than the flux calculated over the total surface area, if all of the heat flows through the spots. A single spot may then be treated by including with it a semi-infinite rod of cross-sectional area N^{-1}. The problem is idealized as follows:

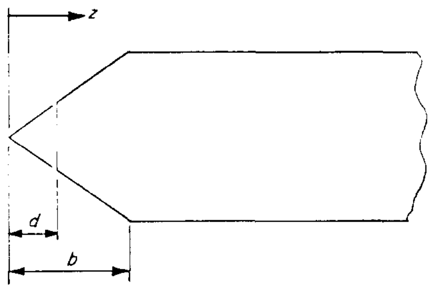

FIG. 2.

The response is to be found for a semi-infinite rod of the shape shown in Fig. 2, which is at a uniform temperature of zero initially and is then heated to a temperature T_0 at the surface $z = d$. The cross-sectional area, A, increases from d to b, in such a way that:

$$A = (\gamma z)^2 \tag{15}$$

where γ is a constant to be evaluated below and,

$$d^2/b^2 = (NA_0). \tag{16}$$

This model does not necessarily assume that the surface profile is given by the above expression, but only that the heat flows through a path which can be approximated in this way.

The temperature distribution is found from:

$$\left.\begin{aligned} &\frac{\partial^2 V}{\partial z^2} + \frac{2}{z}\frac{\partial V}{\partial z} = \frac{1}{\alpha}\frac{\partial V}{\partial t}\, d < z < b \\ &\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial z}; \quad b < z \end{aligned}\right\} \tag{17}$$

and the boundary conditions

$$\left.\begin{aligned} &T = V = 0; && t = 0, d < z \\ &V = T_0; && t > 0, z = d \\ &V = T; && t > 0, z = b \\ &\frac{\partial V}{\partial z} = \frac{\partial T}{\partial z}; && t > 0, z = b \end{aligned}\right\} \tag{18}$$

Once the response to this step increase in temperature at $z = a$ is found, formulations for other boundary conditions may easily be established since the system is linear. The transformed equations are:

$$\frac{1}{z}\frac{d^2(zV_s)}{dz^2} - \frac{s}{\alpha}V_s = 0; \quad d < z < b \tag{19}$$

$$\frac{d^2 T_s}{dz^2} - \frac{s}{\alpha}T_s = 0; \quad b < z. \tag{20}$$

A solution of (19) which satisfies (18) is:

$$V_s = \frac{T_0 d}{sz}\cosh\sqrt{\left(\frac{s}{\alpha}\right)}(z-d) + \frac{A}{z}\sinh\sqrt{\left(\frac{s}{\alpha}\right)}(z-d). \tag{21}$$

And a solution of (20) which goes to zero at infinity is

$$T_s = B\exp\left[-\sqrt{\left(\frac{s}{\alpha}\right)}(z-b)\right]. \tag{22}$$

Using the other boundary conditions to evaluate A and B gives

$$T_s = \frac{T_0 d \exp\left[-\sqrt{\left(\frac{s}{\alpha}\right)}(z-b)\right]}{sb\left[\sinh\sqrt{\left(\frac{s}{\alpha}\right)}(b-d) + \cosh\sqrt{\left(\frac{s}{\alpha}\right)}(b-d) - \frac{1}{b\sqrt{(s/\alpha)}}\sinh\sqrt{\left(\frac{s}{\alpha}\right)}(b-d)\right]} \tag{23}$$

and

$$V_s = \frac{T_0 d}{sz}\left\{\frac{\exp\left[-\left(\frac{s}{\alpha}\right)(b-z)\right] + \left[2\sqrt{\left(\frac{s}{\alpha}\right)}b - 1\right]\exp\left[\sqrt{\left(\frac{s}{\alpha}\right)}(b-z)\right]}{\exp\left[-\sqrt{\left(\frac{s}{\alpha}\right)}(b-d)\right] + \left[2\sqrt{\left(\frac{s}{\alpha}\right)}b - 1\right]\exp\left[\sqrt{\left(\frac{s}{\alpha}\right)}(b-d)\right]}\right\}. \tag{24}$$

The transform of the heat flux at $z = d$ is:

$$q'(d, s) = -K\frac{\partial V_s}{\partial z}\bigg|_{z=d} = \frac{T_0 K}{sd} + \frac{T_0 K}{\sqrt{(sa)}}\left\{\frac{2b\sqrt{\left(\frac{s}{a}\right)} - 1 - \exp\left[-2\sqrt{\left(\frac{s}{a}\right)}(b-d)\right]}{2b\sqrt{\left(\frac{s}{a}\right)} - 1 + \exp\left[-2\sqrt{\left(\frac{s}{a}\right)}(b-d)\right]}\right\}. \quad (25)$$

For short values of time, the inverse of the above equation is found to be approximately

$$q'(a, t) = KT_0\left(\frac{1}{\sqrt{(\pi a t)}} + \frac{1}{d}\right) \quad (26)$$

which agrees with (13a) since T_0 in this case is one half the initial temperature difference between two like materials.

For long values of time a solution is found by expanding the denominator of (23) in powers of s. If only two terms are kept, the result is:

$$T_s = \frac{T_0 \frac{d}{b(b-d)} \exp\left[-\sqrt{\left(\frac{s}{a}\right)}(z-b)\right]}{s\left[\sqrt{\left(\frac{s}{a}\right)} + \frac{d}{b(b-d)}\right]}. \quad (27)$$

This result is exactly analogous to the solution of the problem of heating a semi-infinite body through a surface coefficient, H, which has for a transformed solution

$$T_s = \frac{T_0 H \exp\left[-\sqrt{\left(\frac{s}{a}\right)}x\right]}{sK\left[\sqrt{\left(\frac{s}{a}\right)} + \frac{H}{K}\right]}. \quad (28)$$

Therefore at sufficiently long times,

$$t > \frac{b^2(b-d)^2}{d^2 a},$$

the constriction resistance behaves as a surface resistance equal to

$$R = \frac{1}{H} = \frac{b(b-d)}{dK}. \quad (29)$$

For contact of two bodies the total resistance is the sum of the individual resistance, so that

$$R = \frac{b_1(b_1 - d_1)}{d_1 K_1} + \frac{b_2(b_2 - d_2)}{d_2 K_2} \quad (30)$$

or since the geometry can be taken to be similar in each body

$$R = \frac{b(b-d)}{d}\left(\frac{K_1 + K_2}{K_1 K_2}\right). \quad (30a)$$

The average spacing, δ, between contact spots is

$$\delta = \frac{1}{\sqrt{N}} = \gamma b \quad (31)$$

where γ is a constant on the order of one. Then using

$$d = b\sqrt{(NA_0)} = \frac{\sqrt{A_0}}{\gamma} \quad (15a)$$

it is found that

$$R = \frac{(1 - \sqrt{(NA_0)})(K_1 + K_2)}{\gamma\sqrt{N}\sqrt{(NA_0)} \cdot K_1 K_2} \quad (32)$$

or

$$H = \frac{\gamma\sqrt{N}\sqrt{(NA_0)}K_1 K_2}{[1 - \sqrt{(NA_0)}](K_2 + K_1)} \quad (32a)$$

if

$$\sqrt{(NA_0)} \ll 1$$

that is, if only a small fraction of the surfaces actually touch, then

$$R = \frac{K_2 + K_1}{N(\sqrt{A_0})K_1 K_2 \gamma} \quad (32b)$$

or per spot

$$\rho = \frac{K_2 + K_1}{\sqrt{(\pi a)}K_1 K_2 \gamma} \quad (33)$$

where a is the radius of the contact spot. This result agrees in form and magnitude with that derived above for a single spot in an infinite contact plane and shows that if the spots are small and far apart, the resistance they offer to heat flow is not appreciably affected by the overlap of adjacent temperature fields because the major part of the resistance lies very close to each spot. The value of the constant γ is thus about two.

If another term is kept in the denominator of equation (23),

$$T_s = \frac{T_0 \dfrac{d}{b(b-d)} \exp\left[-\sqrt{\left(\dfrac{s}{a}\right)}(z-b)\right]}{\left[\sqrt{\left(\dfrac{s}{a}\right)} + \dfrac{d}{b(b-d)} + \dfrac{s}{a}\dfrac{(2b+d)(b-d)}{6b}\right]}. \quad (34)$$

The additional term accounts for the heat capacity of the restrictions. Or

$$T_s = \frac{T_0 h_1 \exp\left[-\sqrt{\left(\dfrac{s}{a}\right)}(z-b)\right]}{\left[\sqrt{\left(\dfrac{s}{a}\right)} + h_1 + \dfrac{s}{a}\dfrac{1}{2h_2}\right]} \quad (34a)$$

where

$$\left.\begin{aligned} h_1 &= \frac{b(b-d)}{d} \\ h_2 &= \frac{3b}{(2b+d)(b-d)}. \end{aligned}\right\} \quad (35)$$

Rearranging gives

$$T_s = \frac{T_0 h_1 \exp\left[-\sqrt{\left(\dfrac{s}{a}\right)}(z-b)\right]}{s\sqrt{\left(1-\dfrac{2h_1}{h_2}\right)}}$$

$$\left\{\frac{1}{\left[\sqrt{\left(\dfrac{s}{a}\right)} + h_2 - h_2\sqrt{\left(1-\dfrac{2h_1}{h_2}\right)}\right]} - \frac{1}{\left[\sqrt{\left(\dfrac{s}{a}\right)} + h_2 + h_2\sqrt{\left(1-\dfrac{2h_1}{h_2}\right)}\right]}\right\}. \quad (34b)$$

In the limit as $h_2 \to \infty$, the above equation approaches (27). Letting

$$\left.\begin{aligned} u &= h_2 + h_2\sqrt{\left(1-\frac{2h_1}{h_2}\right)} \\ v &= h_2 - h_2\sqrt{\left(1-\frac{h_2}{2h_1}\right)} \end{aligned}\right\} \quad (36)$$

results in

$$T_s = \frac{2T_0 h_1 h_2}{s(u-v)}\left\{\frac{1}{\sqrt{\left(\dfrac{s}{a}\right)}+v} - \frac{1}{\sqrt{\left(\dfrac{s}{a}\right)}+u}\right\}\exp\left[-\sqrt{\left(\frac{s}{a}\right)}(z-b)\right] \quad (34c)$$

which has the inverse

$$\frac{T_2}{T_0} = \frac{h_1}{\sqrt{\left(1-\dfrac{2h_1}{h_2}\right)}}\left\{\frac{1}{v}\operatorname{erfc}\frac{(z-b)}{2\sqrt{(at)}} - \frac{1}{v}\exp[at\,v^2 + v(z-b)]\operatorname{erfc}\left[v\sqrt{(at)} + \frac{(z-b)}{2\sqrt{(at)}}\right]\right.$$

$$\left. - \frac{1}{u}\operatorname{erfc}\frac{(z-b)}{2\sqrt{(at)}} + \frac{1}{u}\exp[at\,u^2 + u(z-b)]\operatorname{erfc}\left[u\sqrt{(at)} + \frac{(z-b)}{2\sqrt{(at)}}\right]\right\}. \quad (37)$$

This approximate solution to (17) holds for $t > 1/u^2\alpha$. Using the same approximations for a and b, it is found that

$$h_2 = \frac{3\gamma\sqrt{N}}{2 - \sqrt{(NA_0)} - NA_0} \quad (38)$$

$$\frac{2h_1}{h_2} = \frac{4\sqrt{(NA_0)}}{3} + \frac{2NA_0}{3}. \quad (39)$$

For $NA_0 \ll 1$, i.e. a small fractional density of surface contact, which is the case for metal to metal joints, $h_2 \gg h_1$, and

$$\left.\begin{aligned} u &\simeq 2h_2 \\ v &\simeq h_1 \end{aligned}\right\} \quad (40)$$

and

$$\frac{T_2}{T_0} = \frac{2h_2}{2h_2 - h_1}\left\{\operatorname{erfc}\frac{(z-b)}{2\sqrt{(\alpha t)}} - \exp\left[h_1(z-b) + \alpha t h_1^2\right]\operatorname{erfc}\left[\frac{(z-b)}{2\sqrt{(\alpha t)}} + h_1\sqrt{(\alpha t)}\right\} - \frac{h_1}{2h_2 - h_1}\left\{\operatorname{erfc}\frac{(z-b)}{2\sqrt{(\alpha t)}} - \exp\left[2h_2(z-b) + 4\alpha t h_2^2\right]\operatorname{erfc}\left[\frac{(z-b)}{2\sqrt{(\alpha t)}} + 2h_2\sqrt{(\alpha t)}\right]\right\}. \quad (41)$$

From this equation it is seen that the time constants

$$t_1 = \frac{1}{h_1^2\alpha} \quad \text{and} \quad t_2 = \frac{1}{4h_2^2\alpha} \quad (42)$$

govern the response and that for times on the order of t_1, the terms which include h_2 are insignificant provided $NA_0 \ll 1$.

APPLICATIONS

It is shown above that the contact spots act as both heat-flow resistances and capacitances and that under certain conditions the capacitance can be neglected. If the capacitance is negligible the normal convection boundary condition is used for each body at the contacting surface, as

$$K_1\frac{\partial T_1}{\partial X} = K_2\frac{\partial T_2}{\partial X} = H(T_1 - T_2) \quad (43)$$

where H includes both the constriction resistance and any other surface barrier present, such as scale or a gas gap. Care must be taken to insure that these resistances are added in the proper way (i.e. in series or in parallel).

If solutions are desired for short values of time, the capacitance is not negligible, and each boundary condition takes the form

$$\frac{\partial T_1}{\partial X} - \frac{H}{K_1}(T_1 - T_2) - \frac{\omega}{\alpha_1}\frac{\partial T_1}{\partial t} = 0 \quad (44)$$

where

$$\omega = \frac{2 - \sqrt{(NA_0)} - NA_0}{6\gamma\sqrt{(N)}} \quad (45)$$

and accounts for the heat capacity of the restrictions.

If the heat flux at the surface is given and the capacitance is negligible, then the problem can be treated in the ordinary way and a term added to account for the temperature gradient through the restrictions to find the maximum or minimum surface temperature. If the capacitance is not negligible, two equations must be solved for each body, a partial differential equation for the body and an ordinary one for the restrictions. Thus,

$$\frac{\partial^2 T}{\partial X^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}; \quad x > 0 \quad (46)$$

$$K\frac{\partial T}{\partial X} = H(T - V); \quad X = 0 \quad (47)$$

$$K\frac{\omega}{\alpha}\frac{\mathrm{d}V}{\mathrm{d}t} = H(T - V) + f(t); \quad X = 0 \quad (48)$$

where $f(t)$ is the prescribed heat input from friction or some other source. The value of V is then the peak surface temperature. The average surface temperature lies between $V(t)$ and $T(0, t)$.

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Résumé—On trouve des solutions approchées dans différents domaines temporels pour le probléme des flux de chaleur transitoire entre des solides en contact. Les paramètres gouvernant le flux de chaleur et l'applicabilité des différentes solutions sont données en relation avec la géométrie du contact.

Zusammenfassung—Für das Problem des instationären Wärmeflusses in sich berührenden Festkörpern wurden Näherungsgleichungen für verschiedene Zeitabschnitte angegeben. Die, für den Wärmefluss und die Anwendbarkeit der verschiedenen Lösungen massgebenden Parameter sind, abhängig von der Art Ihrer Beziehung, wiedergegeben.

Аннотация—Для задачи определения теплового потока между соприкасающимися твердыми телами найдены приближенные решения, справедливые для различных временных интервалов. Параметры, описывающие тепловой поток, и применимость различных решений даны в зависимости от геометрии соприкасающихся поверхностей.